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FEBRUARY 27TH, 1854.

THOMAS ROMNEY ROBINSON, D.D., PRESIDENT,
in the Chair.

THE Secretary read a paper, by George James Knox, Esq., on the transmutation of metals.

Mr. Thomas Grubb read the following account of a new method of determining, approximately, the spherical aberration of a combination of lenses for microscopic purposes:—

"The methods hitherto at our option for investigating the spherical aberrations of a system of lenses, having spherical surfaces, are—firstly, the purely mathematical, involving (where the thickness of the lenses is required to be included, and more especially where the angle of aperture is considerable) such intricacy in the calculations as renders the process nearly useless to practical persons; and, secondly, the more practical method of constructing diagrams of large size, in which two or more rays, at different distances from the axis of the compound, are geometrically traced, according to the laws of optics.

"Much of my leisure time has, for several years, been devoted to inquiries including, necessarily, the construction of such diagrams; and I cannot speak too highly of their value in giving to the experimenter in optics a thorough practical insight into the effects of the various forms and combinations which will be suggested, more especially when the improvement of the compound objective of the microscope is under consideration.

"It may be desirable here to state, that these diagrams were usually drawn on a scale ten times that intended for use, the radii of the arcs, from which the sines were measured (and which were constant for all the surfaces of a combination), were usually three and sometimes six inches ; the arcs were struck and the sines measured at both sides of the centre (to avoid errors of eccentricity), and the measurement of the sines taken with a scale of fiftieths of an inch, using a magnifier, and estimating to tenths of divisions, or say 1-500th of an inch.

" By tracing—with these precautions, and with care—two rays, one passing through nearly the margin of the combination, and the other at one-half that distance from the axis, an approximation to the state of the spherical aberration of the compound, adequate for such combinations as are used in the construction of the objective of the microscope, may be obtained.

" Some practice in delicate manipulation is, however, required to arrive at the precision indicated, and even for approximate investigations a more accurate method would be desirable.

" When, however, combinations of small aperture, compared with their foci (for example, such proportions of these as are adapted to the object glass of the telescope), are under examination, the method of diagram fails entirely in the accuracy requisite to estimate the correction of the spherical aberration.

" Such difficulties in the way of practical research induced me to consider how far the objections, arising out of the intricacy of the one mode of investigation, and the inadequacy of the other, might be obviated by adopting a *mixed* method of proceeding. In short, it appeared that the diagrams not only included those conditions arising out of the thickness of the lenses, but that they would serve to furnish with sufficient accuracy for all cases that point in each surface where the ray impinges, and also the angle of its incidence. Little more thus remained to be done than to find a simple expression for the aberration of each surface for the ray, and this, after some trials, I obtained in a very convenient form, as follows:—

"*D* being the deviation of the ray (not from one lens into another, but always) either from air into a lens, or from a lens into air, *a* is as ($\sin^2 D$). Then, calling the aberration of the entire combination *A*, and a_1, a_2, a_3, &c., the aberrations (calculated from the above formula) at the 1st, 2nd, 3rd, &c., surfaces of the compound, *A* is equal to the sum of a_1, a_2, a_3, &c. (respect being paid to their signs), these being plus when the ray is deflected towards the axis, and minus when the contrary.

"When the spherical aberration of the compound under examination is corrected, the sums of the + and − aberrations will be equal, and *A* will evidently be zero, while in any other case, the + and − quantities will exhibit a fraction showing their relative proportion, and will indicate to the practical operator the means of a closer approximation to perfect correction.

"The entire process may be shortly described as follows:—

"A diagram of the combination to be examined being made with care, and of a size as large as circumstances permit,—one single ray must be traced (in the usual manner) through the diagram; the most suitable distance for this ray from the axis of the combination will depend upon the angular aperture. For combinations of small angular aperture I prefer 3-4ths to 4-5ths of the semi-aperture for this distance. In addition to this (should the combination have any two surfaces in contact), the direction of the ray from these *into air* must be projected.

"The diagram is now fitted for obtaining the sine of *D* for each surface; these, being measured on a scale of equal parts, and tabulated, and afterwards squared, give a_1, a_2, a_3, &c., which being separated, as directed, into + and −, give, in their sums, two quantities, representing respectively the positive and negative aberration of the whole combination.

"Having examined diagrams made some time ago, and the calculations connected with same, with a view to ascertain the limits of error of this process, it appears that, allowing an error equal to 2-10ths of a division of the scale used in estimating the sines, the probable limits of error are only 1-50th part of the numerical sum of the aberrations, a quantity which may be considered insensible in practice, and probably not one-half of that error, or departure from the true spherical figure of a surface, which takes place (and with contrary signs) during the polishing, according as the lens, or polisher, is upper during the process."

Rev. Dr. Graves read a note from Sir W. R. Hamilton, in which he stated that he had lately arrived at a variety of results respecting the integrations of certain equations, which might not be unworthy of the acceptance of the Academy, and the investigation of which had been suggested to him by Mr. Carmichael's printed Paper, and by a manuscript which he had lent Sir W. Hamilton, who writes,—"In our conclusions we do not quite agree, but I am happy to acknowledge my obligations to his writings for the suggestions above alluded to, as I shall hereafter more fully express.

"So long ago as 1846, I communicated to the Royal Irish Academy a transformation which may be written thus (see the Proceedings for the July of that year):

$$D_x^2 + D_y^2 + D_z^2 = -(iD_x + jD_y + kD_z)^2; \qquad (1)$$

and which was obviously connected with the celebrated equation of Laplace.

"But it had quite escaped my notice that the principles of quaternions allow also this other transformation, which Mr. Carmichael was the first to point out:

$$D_z^2 + D_x^2 + D_y^2 = (D_z - iD_x - jD_y)(D_z + iD_x + jD_y). \qquad (2)$$

And therefore I had, of course, not seen, what Mr. Carmichael has since shown, that the integration of Laplace's equation of